Knowledge Discovery and Data Mining

Unit # 6

Acknowledgement

- Most of the slides in this presentation are taken from course slides provided by
  - Han and Kimber (Data Mining Concepts and Techniques) and
  - Tan, Steinbach and Kumar (Introduction to Data Mining)

Bayes Theorem

- $P(A \mid B) = \dfrac{P(B \mid A)\,P(A)}{P(B)}$

  $= \dfrac{P(B \mid A)\,P(A)}{P(B|A)P(A)+P(B|\neg A)P(\neg A)}$
- P(A) is the prior probability and P(A | B) is the posterior probability.
- Suppose events $A_1, A_2, \ldots\ldots, A_k$ are mutually exclusive and exhaustive; i.e., exactly one of the events must occur. Then for any event B:

  $P(A_i \mid B) = \dfrac{P(B \mid A_i)\,P(A_i)}{\sum P(B \mid A_i)\,P(A_i)}$

Example I

- According to American Lung Association, 7% of the population has lung cancer. Of these people having lung disease, 90% are smokers; and of those not having lung disease, 25.3% are smokers.
- Determine the probability that a randomly selected smoker has lung cancer.

Bayesian Classifiers

- Consider each attribute and class label as random variables
- Given a record with attributes $(A_1, A_2, \ldots, A_n)$
  - Goal is to predict class C
  - Specifically, we want to find the value of C that maximizes $P(C \mid A_1, A_2, \ldots, A_n)$
- Can we estimate $P(C \mid A_1, A_2, \ldots, A_n)$ directly from data?

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Bayesian Classifiers

- Approach:
  - compute the posterior probability $P(C \mid A_1, A_2, \ldots, A_n)$ for all values of C using the Bayes theorem

$$\boxed{P(C \mid A_1 A_2 \ldots A_n) = \frac{P(A_1 A_2 \ldots A_n \mid C) P(C)}{P(A_1 A_2 \ldots A_n)}}$$

  - Choose value of C that maximizes $P(C \mid A_1, A_2, \ldots, A_n)$
  - Equivalent to choosing value of C that maximizes $P(A_1, A_2, \ldots, A_n \mid C) P(C)$
- How to estimate $P(A_1, A_2, \ldots, A_n \mid C)$?

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Naive Bayes

- Naïve Bayes classifiers assume that the effect of an attribute value on a given class is independent of the values of the other attributes.
- This assumption is called class conditional independence.
- It is made to simplify the computations involved and, in this sense, is considered "naïve".

Bayesian Networks

- Bayesian belief networks are graphical models, which unlike naïve Bayesian classifiers, allow the representation of dependencies among subsets of attributes.
- Bayesian belief networks can also be used for classification.

Naïve Bayes Classifier

- Assume independence among attributes A_i when class is given:
  - $P(A_1, A_2, \ldots, A_n | C) = P(A_1 | C_j) P(A_2 | C_j) \ldots P(A_n | C_j)$
  - Can estimate $P(A_i | C_j)$ for all A_i and C_j.
  - New point is classified to C_j if $P(C_j) \prod P(A_i | C_j)$ is maximal.

How to Estimate Probabilities from Data?

Tid	Refund	Marital Status	Taxable Income	Evade
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

- Class: $P(C) = N_c/N$
  - e.g., $P(No) = 7/10$, $P(Yes) = 3/10$
- For discrete attributes:

  $$P(A_i \mid C_k) = |A_{ik}| / N_c$$

  - where $|A_{ik}|$ is number of instances having attribute A_i and belongs to class C_k
  - Examples:

    P(Status=Married|No) = 4/7
    P(Refund=Yes|Yes)=0

Naïve Bayes
Classification: Mammals vs. Non-mammals

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

- Train the model (learn the parameters) using the given data set.
- Apply the learned model on new cases.

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Naïve Bayes
Classification: Mammals vs. Non-mammals

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
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salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

A: attributes

M: mammals

N: non-mammals

$$P(A|M) = \frac{6}{7} \times \frac{6}{7} \times \frac{2}{7} \times \frac{2}{7} = 0.06$$

$$P(A|N) = \frac{1}{13} \times \frac{10}{13} \times \frac{3}{13} \times \frac{4}{13} = 0.0042$$

$$P(A|M)P(M) = 0.06 \times \frac{7}{20} = 0.021$$

$$P(A|N)P(N) = 0.004 \times \frac{13}{20} = 0.0027$$

P(A|M)P(M) > P(A|N)P(N)

=> Mammals

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Example: Play Tennis

Outlook	Temperature	Humidity	Windy	Class
sunny	hot	high	false	N
sunny	hot	high	true	N
overcast	hot	high	false	P
rain	mild	high	false	P
rain	cool	normal	false	P
rain	cool	normal	true	N
overcast	cool	normal	true	P
sunny	mild	high	false	N
sunny	cool	normal	false	P
rain	mild	normal	false	P
sunny	mild	normal	true	P
overcast	mild	high	true	P
overcast	hot	normal	false	P
rain	mild	high	true	N

P(P) = 9/14
P(N) = 5/14

Outlook	Temperature	Humidity	Windy	Class
rain	hot	high	false	?

outlook	
P(sunny|p) = 2/9	**P(sunny|n) = 3/5**
P(overcast|p) = 4/9	**P(overcast|n) = 0**
P(rain|p) = 3/9	**P(rain|n) = 2/5**
temperature	
P(hot|p) = 2/9	**P(hot|n) = 2/5**
P(mild|p) = 4/9	**P(mild|n) = 2/5**
P(cool|p) = 3/9	**P(cool|n) = 1/5**
humidity	
P(high|p) = 3/9	**P(high|n) = 4/5**
P(normal|p) = 6/9	**P(normal|n) = 2/5**
windy	
P(true|p) = 3/9	**P(true|n) = 3/5**
P(false|p) = 6/9	**P(false|n) = 2/5**

Characteristics of Naïve Bayes Classifiers

- They are robust to isolated noise points because such points are averaged out when estimating conditional probabilities from data.
- Naïve Bayes classifiers can also handle missing values by ignoring the example during model building and classification.
- They are robust to irrelevant attributes. If Xi is an irrelevant attribute, then P(Xi | Y) becomes almost uniformly distributed.
- Correlated attributes can degrade the performance of naïve Bayes classifiers because the conditional independence assumption no longer holds for such attributes.

How Effective are Bayesian Classifiers?

- Various empirical studies of this classifier in comparison to decision tree and neural network classifiers have found it to be compariable in some domain.
- In theory, Bayesian classifiers have the minimum error rate in comparison to all other classifiers.
- However, in practice this is not always the case, owning to inaccuracies in the assumptions made of its use, such as class conditional independence, and the lack of available probability data.